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Exploring Norm-Weightable Riesz Spaces and the Associated Dual Complexity Space

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Abstract

The theory of complexity spaces has been introduced in [Sch95], where the applicability to the complexity analysis of Divide & Conquer algorithms has been discussed. This analysis has been based on the Banach Fixed Point Theorem, which has led to the study of biBanach spaces in [RS98]. In [RS96] we have introduced the dual complexity space as a convenient tool to carry out a mathematical analysis of complexity spaces (cf. also [RS98]). We recall that the complexity space as well as its dual are weightable quasi-metric spaces or, equivalently, partial metric spaces (cf. [Sch95], [RS96] as well as [K¨un93],[KV94] and [Mat94]. Recently it has been shown in [Sch02a] that partial metric spaces correspond dually, in the context of Domain Theory, to semivaluation spaces. Here, we show that the dual complexity space is the negative cone of a biBanach norm-weightable Riesz space (e.g. [BOU52] and [RS98]) and characterize the class of norm-weightable Riesz spaces in terms of semivaluation spaces. In particular, we show that the quasi-norm of an element of such a Riesz space is the quasi-norm of its projection on the negative cone. Hence, quasi-norms are completely determined by partial metrics, justifying, in this context, O' Neill's analogy between these notions. In [Sch02a], it is shown that quasi-uniform semilattices arise naturally in Domain Theory, which motivates a generalization of our characterization to the context of norm-weightable quasi-uniform Riesz spaces.

Background

Throughout this paper the letters R, R+ and ω will denote the set of all real numbers, of all nonnegative real numbers and of all nonnegative integer numbers respectively. A function d: X × X → R+ isa quasi-pseudo-metric on X iff 1 Supported by Science Foundation Ireland, Enterprise Ireland research grant BR/2001/141 and a PhD studentship under the Boole Centre for Research in Informatics, UCC 2 The second author acknowledges the support of the Spanish Ministry of Science and Technology, grant BFM2000-1111 3 The third

author acknowledges the support of Science Foundation Ireland 4 AMS (2000) Subject classification: 54E15, 54E35, 54C35, 46B20,22A26 5 Key words and phrases: Riesz space, quasi-norm, quasi-metric space, norm-weightable, negative cone, dual complexity space, quasi-uniform space, join semilattice.

1) $\forall x \in X.\, d(x,x) = 0$.

2) $\forall x,y,z \in X.\, d(x,y) + d(y,z) \geq d(x,z)$

If d is a quasi-pseudo-metric on X, then the function d^{-1} defined on $X \times X$ by $d^{-1}(x,y) = d(y,x)$, is also a quasi-pseudo-metric on X called *the conjugate of d*.

A *quasi-pseudo-metric space* is a pair (X,d) consisting of a set X together with a quasi-pseudo-metric d on X.

In case a quasi-pseudo-metric space is required to satisfy the T_0-separation axiom, we refer to such a space as a *quasi-metric* space.

In that case, condition 1) and the T_0-separation axiom can be replaced by the following condition:

1') $\forall x,y \in X.\, d(x,y) = d(y,x) = 0 \Leftrightarrow x = y$.

If d is a quasi-(pseudo)-metric on X, then d^s is a (pseudo)metric on X, where $\forall x,y \in X.\, d^s(x,y) = \max\{d(x,y), d(y,x)\}$.

A quasi-pseudo-metric space (X,d) is called *order-convex* if $d(x,z) = d(x,y) + d(y,z)$ whenever $z \leq_d \leq y \leq_d x$.

A quasi-(pseudo)-metric d on X is said to be *bicomplete* if d^s is a complete (pseudo)metric on X [FL82].

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Examples: The function d_1 defined on $\mathbb{R}\times\mathbb{R}$ by $d(x,y)=\max\{y-x,0\}$ is a quasi-metric on $\mathbb{R}$ such that $(d_1)^s$ is the usual metric on $\mathbb{R}$.

The function d_2 defined on $(0,\infty]\times(0,\infty]$ by $d_2(x,y)=\max\{\frac{1}{y}-\frac{1}{x},0\}$ is also a quasi-metric on $(0,\infty]$, where we have adopt the convention that $\frac{1}{\infty}=0$.

The complexity (quasi-metric) space has been introduced in [Sch95] as a part of the development of a topological foundation for the complexity analysis of algorithms. In [RS96] we have introduced the dual complexity (quasi-metric) space as an appropriate tool to carry out a mathematical analysis of complexity spaces (see [RS98]).

We recall that the complexity space (with range $(0,\infty]$) is the pair $(\mathcal{C},d_{\mathcal{C}})$, where $\mathcal{C}=\{f:\omega\to(0,\infty]\mid\sum_{n=0}^{\infty}2^{-n}\frac{1}{f(n)}<\infty\}$ and $d_{\mathcal{C}}$ is the quasi-metric defined on $\mathcal{C}$ by $d_{\mathcal{C}}(f,g)=\sum_{n=0}^{\infty}2^{-n}\max\{\frac{1}{g(n)}-\frac{1}{f(n)},0\}$, whenever $f,g\in\mathcal{C}$. $d_{\mathcal{C}}$ is called in [Sch95] "the complexity distance", and intuitively measures relative improvements in the complexity of programs.

The dual complexity space (with range $\mathbb{R}^+$) is introduced in [RS96] as a pair $(\mathcal{C}^*,d_{\mathcal{C}^*})$, where $\mathcal{C}^*=\{f:\omega\to\mathbb{R}^+\mid\sum_{n=0}^{\infty}2^{-n}f(n)<\infty\}$ and $d_{\mathcal{C}^*}$ is the quasi-metric defined on $\mathcal{C}^*$ by $d_{\mathcal{C}^*}(f,g)=\sum_{n=0}^{\infty}2^{-n}\max\{g(n)-f(n),0\}$, whenever $f,g\in\mathcal{C}^*$.

$(\mathcal{C},d_{\mathcal{C}})$ is isometric to $(\mathcal{C}^*,d_{\mathcal{C}^*})$ by the isometry $\Psi:\mathcal{C}^*\to\mathcal{C}$, defined by $\Psi(f)=1/f$ (see [RS96]).

A quasi-metric space (X,d) is *weightable* iff there exists a function $w\colon X\to\mathbb{R}^+$ such that $\forall x,y\in X.\, d(x,y)+w(x)=d(y,x)+w(y)$. The function w is called a *weighting function*, $w(x)$ is the *weight* of x and the quasi-metric d is *weightable by the function* w. A *weighted* space is a triple (X,d,w) where (X,d) is a quasi-metric space weightable by the function w. A weighting function of a weighted quasi-metric space is *fading* iff the space has points of arbitrary small weight.

We recall that the weighting functions of a weightable quasi-metric space are generated by a unique fading weighting f (e.g. [KV94] or [Sch02a]) in the sense that each weighting is of the form $f+c$ for some real number $c\geq 0$.

Examples: The quasi-metric space $(\mathbb{R}^+,d_1)$ is weightable by the identity function, $w_1(x)=x$. The quasi-metric space $((0,\infty],d_2)$ is weightable by the function $w_2(x)=\frac{1}{x}$. The complexity space $(\mathcal{C},d_{\mathcal{C}})$ is weightable by the function $w_{\mathcal{C}}$ where $\forall f\in\mathcal{C}.\, w_{\mathcal{C}}(f)=\sum_n\frac{2^{-n}}{f(n)}$. The dual complexity space $(\mathcal{C}^*,d_{\mathcal{C}^*})$ is weightable by the function $w_{\mathcal{C}^*}$ where $\forall f\in\mathcal{C}^*.\, w_{\mathcal{C}^*}(f)=\sum_n 2^{-n}f(n)$.

We recall the following definition from [Sch96].

Definition 1.1 If (X,d) is a quasi-metric space then (X,d) is upper weightable iff there exists a weighting function w for (X,d) such that $\forall x,y\in X.\, d(x,y)\leq w(y)$. We refer to such a function w as an upper weighting function. A weighted space (X,d,w) is upper weighted iff w is an upper weighting function.

Examples: The quasi-metric space $(\mathbb{R}^+,d_1)$ is upper weightable by the function w_1, the quasi-metric space $((0,\infty],d_2)$ is upper weightable by the function w_2, the complexity space $(\mathcal{C},d_{\mathcal{C}})$ is upper weightable by the function $w_{\mathcal{C}}$, while the dual complexity space $(\mathcal{C}^*,d_{\mathcal{C}^*})$ is upper weightable by the function $w_{\mathcal{C}^*}$.

A *quasi-uniform space* is a pair $(X,\mathcal{U})$ consisting of a set X with a filter $\mathcal{U}$ on $X\times X$ such that

1) $\forall U\in\mathcal{U}.\,\Delta\subseteq U$
2) $\forall U\in\mathcal{U}\,\exists V\in\mathcal{U}.\,V\circ V\subseteq U$.

In that case, $\mathcal{U}$ is called a *quasi-uniformity* on X and its elements are referred to as *entourages*. The *preorder associated* with a quasi-uniform space $(X,\mathcal{U})$ is the relation $\leq_{\mathcal{U}}$ defined to be the intersection of all the entourages of $\mathcal{U}$.

A subfamily $\mathcal{B}$ of a quasi-uniformity $\mathcal{U}$ is a *base* for $\mathcal{U}$ if each entourage contains a member of $\mathcal{B}$.

The *quasi-uniformity* $\mathcal{U}_d$ *generated by a quasi-pseudo-metric* d on a set X is the filter generated on $X\times X$ by the set of relations $(B_{\epsilon>0})_\epsilon$, where $\forall\epsilon>0.\,B_\epsilon=\{(x,y)\mid d(x,y)<\epsilon\}$. Two quasi-pseudo-metrics are *equivalent* iff they generate the same quasi-uniformity. Two quasi-pseudo-metric spaces are equivalent iff their quasi-pseudo-metrics are equivalent.

The topology $\mathcal{T}(\mathcal{U})$ associated to a quasi-uniformity $\mathcal{U}$ on a set X is the topology generated by the neighbourhood filter base $\mathcal{U}[x]=\{U[x]\mid U\in\mathcal{U}\}$, where $\forall x\in X\,\forall U\in\mathcal{U}.\,U[x]=\{y\mid(x,y)\in U\}$.

If $\mathcal{U}$ is a quasi-uniformity on a set X then the *trace* quasi-uniformity $\mathcal{U}|A$ of $\mathcal{U}$ on a subset A of X is defined by: $\mathcal{U}|A=\{U\cap(A\times A)\mid U\in\mathcal{U}\}$.

If $(X,\mathcal{U})$ and $(Y,\mathcal{V})$ are quasi-uniform spaces, then the *product quasi-uniformity* $\mathcal{U}\times\mathcal{V}$ is the set of all binary relations B on $X\times Y$, such that there is a $U\in\mathcal{U}$ and a $V\in\mathcal{V}$ such that for each (x,y) in $X\times Y$, $B[(x,y)]=U[x]\times V[y]$. The topology induced by the product quasi-uniformity is the product topology.

A function $f\colon(X,\mathcal{U})\to(Y,\mathcal{V})$ is *quasi-uniformly continuous* iff $\forall V\in\mathcal{V}\,\exists U\in\mathcal{U}.\,f^2(U)\subseteq V$, where $f^2(U)=\{(f(x),f(y))\mid(x,y)\in U\}$. A *quasi-unimorphism* $f\colon(X,\mathcal{U})\to(Y,\mathcal{V})$ is a bijection such that both f and f^{-1} are quasi-uniformly continuous.

In case the associated preorder of a quasi-pseudo-metric (quasi-uniform) space is a linear preorder we refer to the space as a *linear* quasi-pseudo-metric (quasi-uniform) space.

A *uniform space* is a quasi-uniform space $(X,\mathcal{U})$ which is such that $\forall U\in\mathcal{U}.\,U^{-1}\in\mathcal{U}$. Given a quasi-uniform space $(X,\mathcal{U})$ then the *uniform space associated to* $(X,\mathcal{U})$ is defined to be the space $(X,\mathcal{U}^s)$ where $\mathcal{U}^s=\{V\mid V\subseteq X\times X$ and $\exists U\in\mathcal{U}$ such that $V\supseteq U\cap U^{-1}\}$.

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A *weak quasi-pseudo-metric (weak quasi-uniform) join semilattice* is a quasi-pseudo-metric (quasi-uniform) space which is a join semilattice for its associated preorder. We say that a quasi-pseudo-metric space (X,d) has a maximum $x_0 \in X$ if $x \leq_d x_0$ for all $x \in X$, where $\leq_d$ is the associated preorder of (X,d).

The terminology of *quasi-pseudo-metric (quasi-uniform) (semi)lattice* is reserved for quasi-pseudo-metric (quasi-uniform) spaces which are (semi)lattices for which the operations are quasi-uniformly continuous with respect to the product quasi-uniformity $\mathcal{U}_d \times \mathcal{U}_d$ $(\mathcal{U} \times \mathcal{U})$. This is in accordance with the terminology used for the theory of uniform lattices (e.g. [Web91] and [Web]).

As discussed in [Sch02a], quasi-uniform (semi)lattices arise naturally in Domain Theory and include in particular the class of totally bounded Scott domains discussed in [Smy91], the Baire quasi-metric spaces of [Mat95] as well as the complexity spaces of [Sch95].

Each of these structures turns out to satisfy an "optimality condition", which is tightly related to compactness (cf. [Sch02a]).

An *optimal weak quasi-pseudo-metric join semilattice* is a weak quasi-pseudo-metric join semilattice (X,d) such that $d(x \sqcup y, y) = d(x,y)$ for all $x,y \in X$.

We recall that a quasi-pseudo-metric join semilattice (X,d) is optimal if and only if for all $x,y,z \in X. d(x \sqcup z, y \sqcup z) \leq d(x,y)$ (cf. [Sch97]). We remark that this equivalent condition to optimality is exactly the more familiar notion of $\sqcup$-invariance as discussed in [Gie80]. Hence we obtain that any optimal weak quasi-pseudo-metric join semilattice is a quasi-pseudo-metric join semilattice and we will simply refer to such structures in the following as "optimal quasi-metric join semilattices".

We recall the following useful generalizations of valuations (e.g. [Bir84]) to the context of semilattices, introduced in [Sch02a]:

If $(X,\preceq)$ is a join semilattice then a function $f:(X,\preceq) \to \mathbb{R}^+$ is *join-modular* iff

$$\forall x,y,z \in X.\, f(y \sqcup z) - f(x \sqcup z) \geq f(y) - f(x \sqcup y)$$

and f is *co-join-modular* iff

$$\forall x,y,z \in X.\, f(y \sqcup z) - f(x \sqcup z) \leq f(y) - f(x \sqcup y).$$

A *join valuation* on a join semilattice is a join-modular increasing function on this semilattice. A *join co-valuation* on a join semilattice is a co-join-modular decreasing function on this semilattice.

A real-valued function f on a join semilattice $(X,\sqsubseteq)$ is called *positive* (*negative*) if $\forall x,y \in X. x \sqsubset y \Rightarrow f(x) < f(y) (f(x) > f(y))$.

In the following section we recall the main definitions and results of [RS98] on norm-weightable biBanach spaces.

We provide a brief motivation for the study of biBanach spaces in connection to complexity spaces.

We recall that the complexity analysis of Divide & Conquer algorithms involves functionals on complexity spaces of the following type:

$$\Phi_{\mathcal{E}}(f) = \lambda n.\ \text{if } n = 1 \text{ then } c \text{ else } af(\frac{n}{b}) + h(n).$$

Since these functionals are defined in terms of the pointwise operations of addition and of scalar multiplication, which intuitively reflect operations carried out by the algorithm on the given datastructures, it is natural to equip complexity spaces with corresponding operations. This approach directly leads to the study of (semi)linear spaces.

Also, in [Sch95] the complexity analysis of Divide & Conquer algorithms has been carried out via the Banach Fixed Point Theorem. The version of the Banach Fixed Point Theorem used in [Sch95] however is formulated in terms of bicomplete quasi-metric spaces, rather than in terms of say biBanach spaces, as one might expect.

In the following section, we provide the necessary definitions in order to formulate the new approach via biBanach spaces (cf. [RS98]). We also recall the useful notion of norm-weightedness from [RS98], which will allow us to show that the weight of the dual complexity space is the restriction of a quasi-norm of a biBanach space.

2 Norm-weightable biBanach spaces

An *ordered linear space* is a quadruple $(E,\sqsubseteq,+,\cdot)$ such that $(E,+,\cdot)$ is a linear space, say with neutral element $\mathbf{0}$ and $(E,\sqsubseteq)$ is an order such that

(1) $\forall x,y,z \in X. x \sqsubseteq y \Rightarrow x+z \sqsubseteq y+z$
(2) $\forall x \in E\, \forall \lambda \in \mathbb{R}^+. x \sqsupseteq \mathbf{0} \Rightarrow \lambda x \sqsupseteq \mathbf{0}$.

Remark: In any ordered linear space, conditions (1) and (2) in fact imply conditions (1′) and (2′) obtained from (1) and (2) by replacing the implication by an equivalence (cf. [BOU52]).

An element x of an ordered linear space $(E,\sqsubseteq,+,\cdot)$ is *positive (negative)* iff $x \sqsupseteq \mathbf{0}$ $(x \sqsubseteq \mathbf{0})$, where $\mathbf{0}$ is the neutral element of the linear space.

In our context a *semilinear space* on $\mathbb{R}^+$ is an ordered triple $(E,+,\cdot)$, such that $(E,+)$ is an Abelian semigroup containing the neutral element $\mathbf{0}$, and $\cdot$ is a function from $\mathbb{R}^+ \times \mathbb{E}$ to E such that for all $x,y \in E$ and $a,b \in \mathbb{R}^+$: $a\cdot(b\cdot x) = (ab)\cdot x$, $(a+b)\cdot x = (a\cdot x)+(b\cdot x)$, $a\cdot(x+y) = (a\cdot x)+(a\cdot y)$, and $1\cdot x = x$.

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We recall that every semilinear space is a *cone* in the sense of Keimel and Roth [KR92]. In the context of this paper we use this terminology rather than the one of semilinear spaces (as used in [RS98]). The motivation for this is that in the context of Riesz spaces, the terminology of cones is traditionally used (e.g. [BOU52]).

We remark that for a linear space E (on $\mathbb{R}$) the traditional definition of a cone with top $\mathbf{0}$ is a subset of E which is closed under addition and under positive scalar multiplication. It is easy to verify that in the context of linear spaces the two notions of a cone coincide.

Example: The set of all positive elements of an ordered linear space forms a cone, which we refer to as the *positive cone* of the space. Similarly, the set of all negative elements of an ordered linear space forms a cone, which we refer to as the *negative cone* of the space.

Let $(E,+,\cdot)$ be a linear space on $\mathbb{R}$. A *quasi-norm* on E is a nonnegative real-valued function $\|.\|$ on E such that for all $x,y\in E$ and $a\in\mathbb{R}^+$:

(i) $\|x\| = \|-x\| = 0 \Leftrightarrow x = \mathbf{0}$ (where $\mathbf{0}$ denotes the neutral element of $(E,+)$);
(ii) $\|ax\| = a\|x\|$;
(iii) $\|x+y\| \le \|x\| + \|y\|$.

Note that the function $\|.\|^s$ defined on E by $\|x\|^s = \max\{\|x\|, \|-x\|\}$, for all $x\in E$, is a norm on E.

If a quasi-norm $\|\cdot\|$ exists on a linear space E, we say that the linear space is *quasi-normalizable* and refer to the pair $(E,\|.\|)$ as a *quasi-normed linear space*.

The quasi-norm $\|.\|$ induces, in a natural way, a quasi-metric $d_{\|.\|}$ on E, defined by

$$d_{\|.\|}(x,y) = \|y - x\| \text{ for all } x,y\in E.$$

For a given quasi-normed linear space $(E,\|.\|)$, we refer to the order associated to the quasi-metric $d_{\|.\|}$ as the *order associated to the quasi-norm*.

According to [RS98] a *biBanach space* is a quasi-normed linear space $(E,\|.\|)$ such that the induced quasi-metric $d_{\|.\|}$ is bicomplete.

Proof. Let E be a Riesz space with negative cone C^- and let $f: C^- \to \mathbb{R}^+$ be a quasi-norm satisfying (1).

Since f satisfies (1), we obtain in particular that f is decreasing, since $\forall x,y\in C^-. x\sqsupseteq y \Rightarrow 0\le f(y-x) = f(y)-f(x)$ and hence $f(y)\ge f(x)$.

We define $\overline{f}$ by: $\forall x\in E. \overline{f}(x) = f(x^-)$ and verify that $\overline{f}$ satisfies (2) − (4).

To show (2), we remark that, since f is decreasing, $\forall x,y\in E. \overline{f}(x+y) = f((x+y)^-) \le f(x^- + y^-) \le f(x^-) + f(y^-) = \overline{f}(x) + \overline{f}(y)$, where the first inequality follows from Lemma 3.2).

We remark that $\forall x\in E\,\forall a\in\mathbb{R}^+. (ax)^- = ax\sqcap\mathbf{0} = a(x\sqcap\mathbf{0}) = a(x^-)$.

To show (3), we remark that $\forall x\in E\,\forall a\in\mathbb{R}^+. \overline{f}(ax) = f((ax)^-) = f(ax^-) = af(x^-) = a\overline{f}(x)$.

To verify (4), we remark that if $x = \mathbf{0}$ then clearly $x^- = \mathbf{0}$ and $x^+ = \mathbf{0}$ and thus $\overline{f}(x) = f(x^-) = f(\mathbf{0}) = 0$.

Conversely, we assume that $\overline{f}(x) = \overline{f}(-x) = 0$ for some $x\in E$. Then we obtain that $f(x^-) = f((-x)^-) = 0$ and thus $f(x^-) = f(x^+) = 0$. Hence $f(|x|) \le f(x^+) + f(x^-) = 0$. If we assume by contradiction that $x\ne\mathbf{0}$ then in particular $x^-\ne\mathbf{0}$ or $x^+\ne\mathbf{0}$ (since $x = x^- - x^+$). Say w.l.o.g. that $x^-\ne\mathbf{0}$ and thus $x^-\sqsubset\mathbf{0}$. Then, since $|x|\sqsubseteq x^-$, we have $|x|\sqsubset\mathbf{0}$ and thus $f(|x|) > f(\mathbf{0}) = 0$, since f is negative. Thus we obtain a contradiction.

To show the converse, we remark that that $\forall x\in E. f(x) = f(x^- - x^+) = f(x^- - (x^+\sqcup x^-))$ by $(1'')$. Hence $\forall x\in E. f(x) = f(x^- + ((-x^+)\sqcap(-x^-)))$ by Lemma 3.2). So by (f), we obtain that $\forall x\in X. f(x) = f(x^- + ((-x^+) + ((-x^-) - (-x^+))^-)) = f(x^- + ((-x^+) + (-x)^-)) = f(x^- + (-x^+ + x^+)) = f(x^-)$.

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